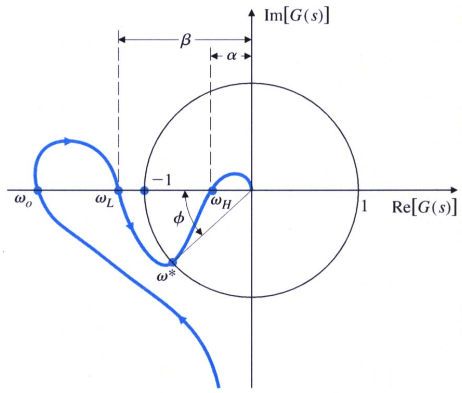
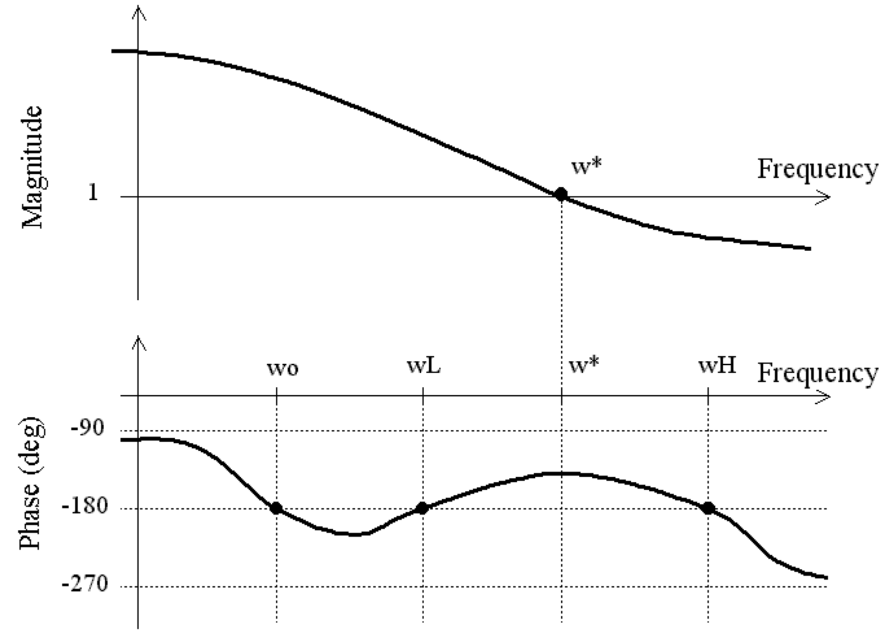
Control Systems : Set 7 : Loopshaping (3) - Solutions

Prob 1 | The Nyquist plot of an open-loop stable system resembles the one shown in the figure below. What are the gain and phase margin(s) for this system given that $\alpha = 0.4$, $\beta = 1.3$ and $\phi = 40°$? Describe what happens to the stability of the system as the gain goes from zero to a very large value. Sketch what the corresponding Bode plot would look like for the system.



Note: What is shown in the figure is only half of the Nyquist plot from $\omega = 0$ to $\omega = \infty$ (i.e., the part that corresponds to the Bode diagram). The full nyquist plot also contains the range $\omega = -\infty$ to $\omega = 0$. However, since the coefficients of the polynomials in our system $G(s)$ are always real numbers, the segment from $[0, \infty]$ will always be a reflection across the real axis of the segment from $[-\infty, 0]$.

The phase margin can be read from the plot as PM = 40°, but now there are several gain margins. If the system gain is increased (multiplied) by $\frac{1}{|\alpha|}$ or decreased (divided) by $|\beta|$, then the system will be unstable.
For very low values of gain, the entire Nyquist plot would be shrunk, and the −1 point would occur to the left of the negative real axis crossing at ω_o, so there would be no encirclements and the system would be stable. As the gain increases, the −1 point occurs between ω_o and ω_L so there is an encirclement and the system is unstable. Further increase of the gain causes the −1 point to occur between ω_L and ω_H so there is no encirclement and the system is stable. Even more increase in the gain would cause the −1 point to occur between ω_H and the origin where there is an encirclement and the system is unstable. The Bode plot would be vaguely like that drawn below:



Magnitude
1
w*
Frequency
Phase (deg)
wo
wL
w*
wH
Frequency
-90
-180
-270

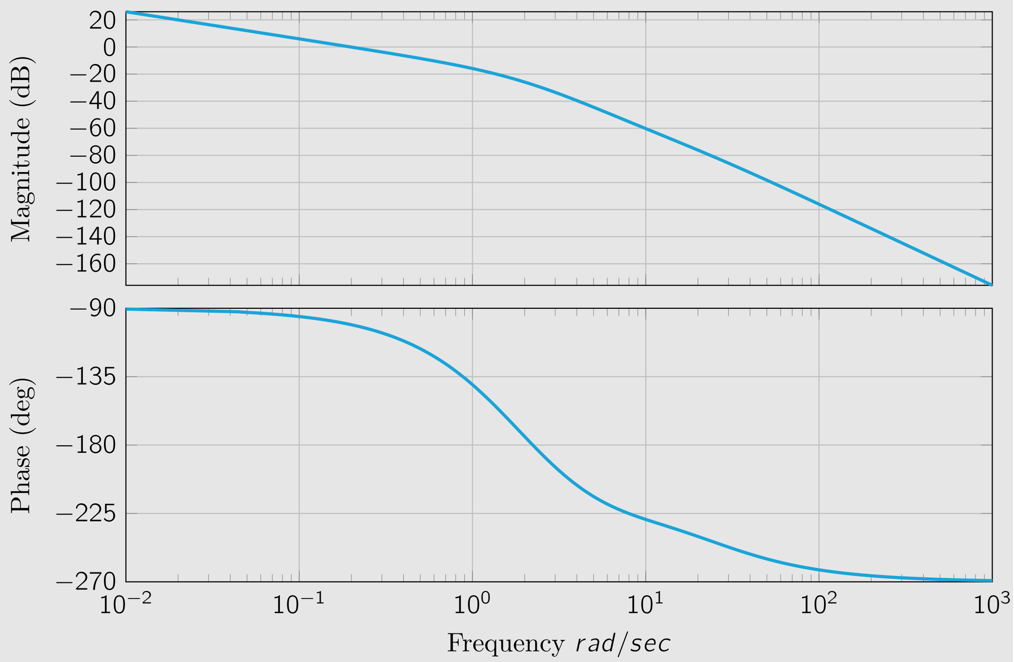
Prob 2 | The steering dynamics of a ship are represented by the transfer function

$$\frac{V(s)}{\delta_r(s)} = G(s) = \frac{K(0.1s+1)}{s(0.5s+1)^2(0.05s+1)}$$

where v is the ship's velocity in meters per second and δ_r is the rudder angle in radians.

a) Use Matlab to generate the Bode plot for $K = 0.2$

```
s = tf('s');
K = 0.2
G = (0.1*s+1) / (s * (0.5*s+1)^2 * (0.05*s + 1))
bode(K*G)
```



b) Indicate the phase margin and the gain margin on the plot

```
[Gm,Pm,Wcg,Wcp] = margin(0.2*G)

Gm = 24.5559
Pm = 79.2547
Wcg = 2.2297
Wcp = 0.1981
```

The closed-loop system would be stable.

c) Is the ship steering system stable for $K = 0.5$

```
[Gm,Pm,Wcg,Wcp] = margin(0.5*G)

Gm = 9.8224
Pm = 64.6990
Wcg = 2.2297
Wcp = 0.4738
```

Yes, the ship would still be stable.

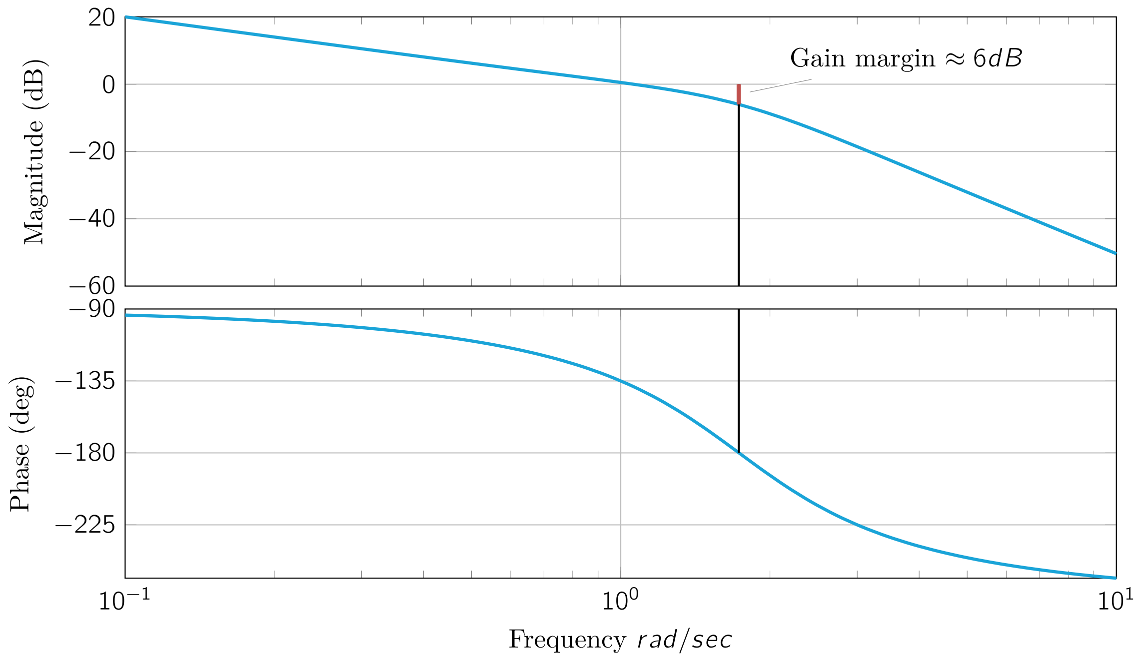
d) What value of K would yield a PM of 30° and what would the crossover frequency be?

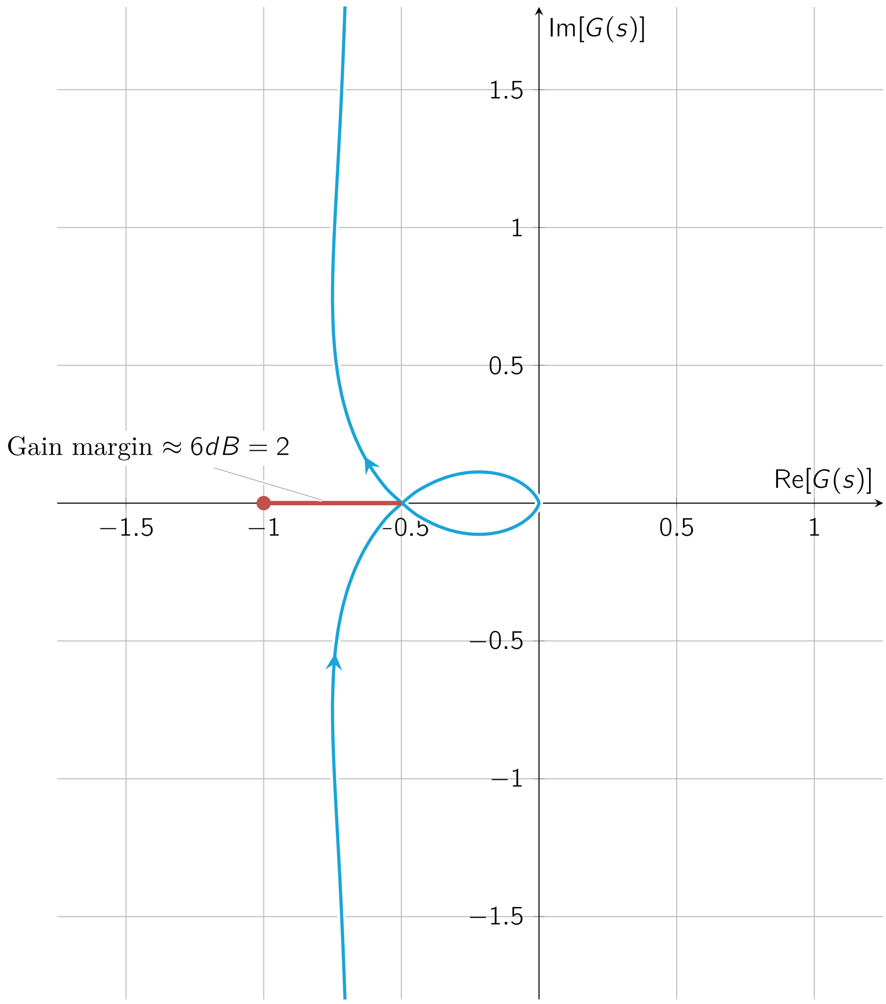
```
[Gm,Pm,Wcg,Wcp] = margin(1.6*G)

Gm = 3.0695
Pm = 31.9328
Wcg = 2.2297
Wcp = 1.1886
```

From the plot we see that a phase of -150 degrees is obtained at a frequency of 1.2 rad/sec, which corresponds to a gain of $|G(j1.2)| = 0.6160$. We set $K = 1/0.616 = 1.6$.

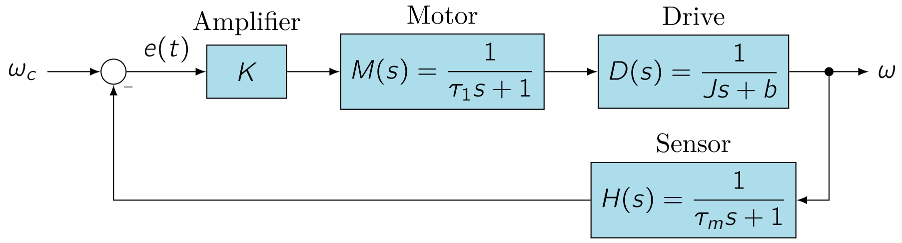
Prob 3 | Consider the Bode plot and the Nyquist plot for the system $G(s)$ below.
Show how the ultimate period (the period of oscillation for Ziegler-Nichols tuning) and the ultimate gain (the gain at which the system oscillates) can be read from the Bode plot and from the Nyquist plot.





The ultimate gain occurs when a proportional controller causes the system to be on the border of unstable, i.e., when the system passes through the Nyquist point. On the Bode plot, we see that this is when the phase is $-180°$, and then the gain is increased/decreased until it passes through 0dB. From the bode plot, we can see that this is the Gain Margin, which is approximately $6dB$. We can also read the gain margin of $2 = 6dB$ from the Nyquist plot as shown.

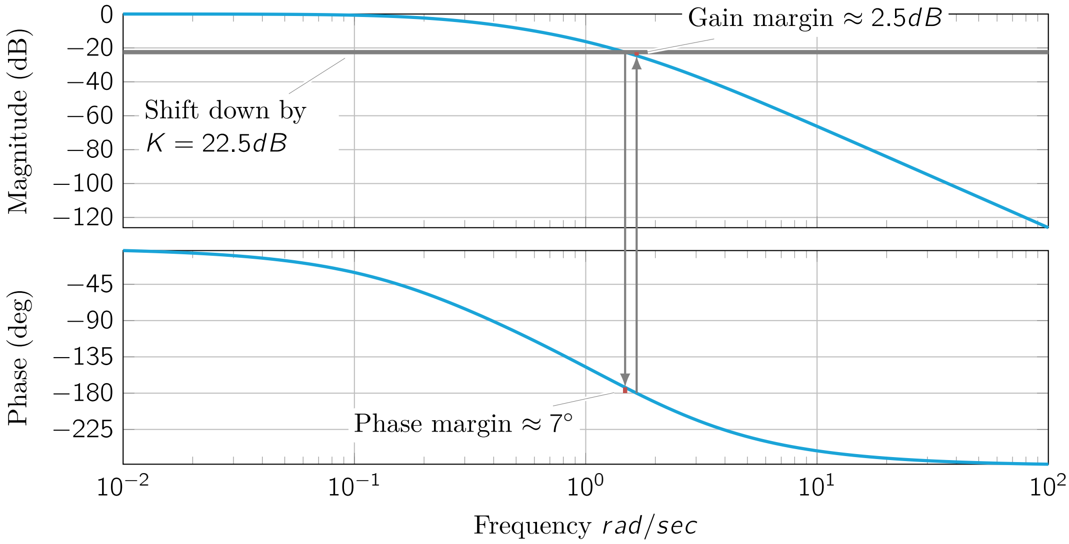
Prob 4 | A speed control system is shown in the figure below. Note that the speed sensor is slow enough that its dynamics mush be included, since the speed-measurement time constant is $\tau_m = 0.5sec$. The time constant of the drive being controlled is $\tau_r = J/b = 4sec$, where the damping constant $b = 1N \cdot m \cdot sec$ and the motor time constant is $\tau_1 = 1sec$.



a) Determine the gain K required to keep the steady-state speed error to less than 7% of the reference speed setting

> This is a Type 0 system, and we want the error in response to a step change in the reference
>
> $$\lim_{t\to\infty} e(t) = \frac{1}{1+\gamma} \geq 0.07$$
> $$\to \gamma \geq 13.3$$
>
> where γ is given by
>
> $$\gamma = \lim_{s\to 0} s^0 \frac{K}{\tau_1 s + 1} \frac{1}{Js + b} \frac{1}{\tau_m s + 1} = \frac{K}{b} = K$$
>
> So we get the result that $K \geq 13.3$ to achieve a tracking error of less than 7%.

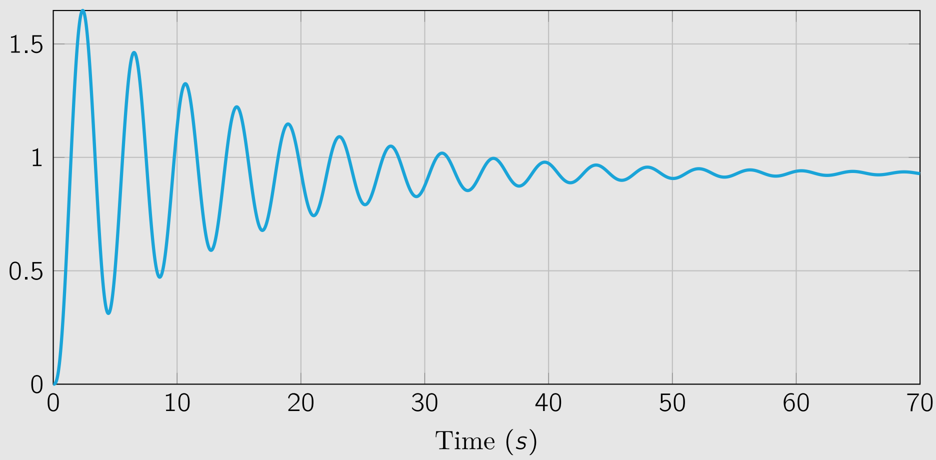
b) Consider the bode plot of the open-loop system $M(s)D(s)H(s)$ show below. Determine the gain and phase margins for the value of K determined above. Is a proportional controller a good design for this system?



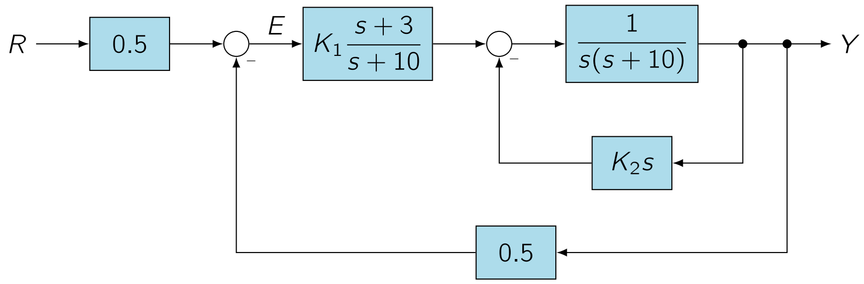
From the bode plot, we can read the GM as approximately $25dB$. However, the control gain K was set to $13.3 = 22.5dB$ in the previous part, and therefore the GM of $KM(s)D(s)H(s)$ is $25 - 22.5 = 2.5dB$.

The phase margin is the phase when the magnitude plot goes through the point $-K = -22.5dB$ (i.e., the zero-dB point of the system $KM(s)D(s)H(s)$). We can see that this is about $7°$.

We can conclude that the phase margin and gain margin for this system are far too low, as can be seen from the step response of the closed-loop system shown below. Therefore, if we want to achieve a tracking error of 7% and better gain and phase margins (better robustness, less oscillation), then we will need to choose a control structure other than a simple proportional gain.

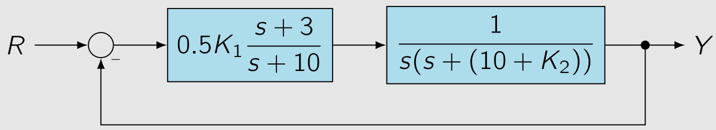


Prob 5 | A block diagram of a control system is shown in the figure below.



a) What is the system type?

We first re-draw the block diagram in a standard form:



We can now see that there is one integrator in the forward path, and therefore this is a Type-1 system.

Alternatively, we could compute the transfer function from the reference to the error, and apply the final value theorem as when we derived the system types in the first place.

b) If R is a step input and the system is closed-loop stable, what is the steady-state tracking error?

The system is Type-1 and so there will be no tracking error for step inputs.

c) What is the steady-state error to a ramp input of velocity 5.0 if $K_2 = 2$ and K_1 is adjusted to give a system step overshoot of 17%? Use Matlab and the step command to determine the value of K_1.

Our first step is to find the value of K_1 that results in an overshoot of 17%. This can be done by simply trying different values of K_1 in Matlab until it looks reasonable. However, we can also write a simple search program (which assumes that the overshoot is monotonic in K_1).

```
% Search range for k1
min_k1 = 1;
max_k1 = 1000;

k2 = 2;
s = tf('s');

while max_k1 - min_k1 > 1e-3
```

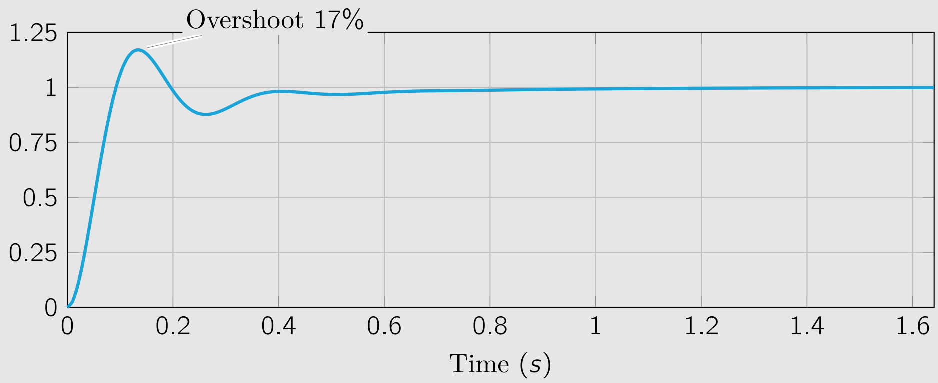
```
  k1 = mean([max_k1 min_k1]);

  % Compute closed-loop system
  G = 1/(s*(s+10));
  K2 = k2*s;
  G_inner = feedback(G,K2);
  K1 = k1*(s+3)/(s+10);
  sys_cl = 0.5*feedback(K1*G_inner,0.5);

  % Compute information about the step response
  dat = stepinfo(sys_cl);
  fprintf('k1 = %5.2f Overshoot = %5.1f%%\n', k1, 100*dat.Overshoot);

  % Reduce the search interval
  if dat.Overshoot < 0.17
    min_k1 = k1;
  else
    max_k1 = k1;
  end
end
```

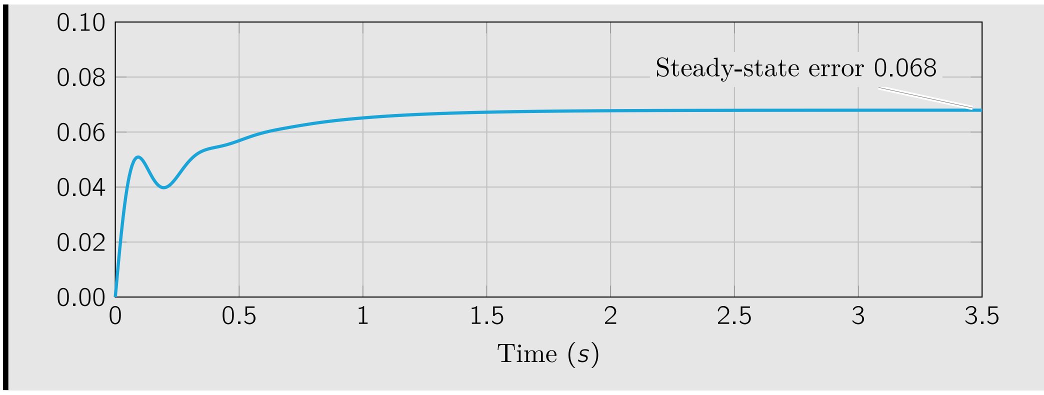
The result is $K_1 = 1177.5$, and the resulting step response is shown below.



We can now compute the velocity constant

$$\begin{aligned}
\gamma &= \lim_{s\to 0} s \cdot 0.5K_1 \frac{s+3}{s+10} \frac{1}{s(s+(10+K_2))} \\
&= 0.5K_1 \frac{3}{10(10+K_2)} \\
&= 0.5 \cdot 1177.5 \frac{3}{10(10+2)} \\
&= 14.7
\end{aligned}$$

We plot below the error in response to a unit ramp input, and see that indeed there is a steady-state error of $1/\gamma = 1/14.7 = 0.068$.


0.10
0.08
0.06
0.04
0.02
0.00
0
0.5
1
1.5
2
2.5
3
3.5
Time (s)
Steady-state error 0.068